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Backward-facing step calculations using the shear improved Smagorinsky model

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Recently a simple modification of the standard Smagorinsky model was proposed that takes into account the physics associated with strong shear in wall-bounded flows. After its initial proposal in Toschi *et al.* (2000) the model was first tested in Leveque *et al.* (2006) in the context of channel flow. Here we test its validity on the more complex backward-facing step geometry. We compare results against experimental data, DNS data, and results obtained with the dynamic model. The model gives accurate results, comparable with those achieved with the dynamic model, at the low Reynolds number of the computations. We identified several issues that should be addressed in order to broaden the applicability of the model to non-stationary flows.

1. Introduction

The prohibitive cost of direct numerical simulations (DNS) of turbulent engineering flows drive the strong interest in simplified models, which are less expensive from a computational point of view, but still relevant in properly describing at least the largest scales of the flow. Furthermore, realistic flowfields are almost always characterized by the presence of walls, often, of quite complex geometry.

Among these simplified models is the Large-Eddy Simulation (LES), in which the large scales of motion, responsible for the momentum and energy transport, are computed, and only the small scales, which are generally more homogeneous and isotropic (and, therefore, easier to model) are parametrized. The effect of the small subgrid scales appears in the governing equations for LES through an additional stress term, which must be modeled. Several subgrid-scale (SGS) models have been proposed in the past (see Sagaut 2002).

Most subgrid-scale models have been derived based on the physics of homogeneous isotropic turbulence. In practice, however, mean shear is important in many applications, especially if solid boundaries are present. The presence of boundaries affects the kinetics of the flow through several mechanisms, one of them being the presence of a strong shear that usually peaks at the boundary and is responsible for the production of streamwise vortices and streaky structures that eventually detach and sustain the turbulent fluctuations in the bulk.

In the present work we test a subgrid-scale model recently formulated in Toschi *et al.* (2000) and Leveque *et al.* (2006) and tested in the context of channel flow. In particular we will focus on the backward-facing step geometry. This case contains significantly more complexities than the standard plane channel flow. In addition to wall effects, in fact, the backward-facing step physics are strongly affected by the detached shear layer, and by the flow in the reattachment region. Adequate resolution of the instability of the shear layer is critical to the prediction of the reattachment point, and to the accuracy

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of the subsequent relaxation region following the reattachment. This case, therefore, is a stringent test both of the numeric and of the models.

The model is presented here, followed by a discussion of results. Finally, some conclusions will be made, and directions for future work will be reviewed.

2. Problem formulation

2.1. *Introduction and definitions*

Here we briefly review the definition and the standard notation for the LES of Navier-Stokes turbulence. As is standard, we consider a filtering operation defined in terms of a convolution kernel G_Δ, applying such kernel to a generic field ϕ will produce a field $\overline{\phi}$, filtered at scale Δ: $\int \phi(\mathbf{x}', t) G_\Delta(\mathbf{x} - \mathbf{x}') d\mathbf{x}'$. The filter width Δ fixes the size of the smallest scale of variation retained in the flow variable $\phi(\mathbf{x}, t)$. Scales smaller than Δ will be averaged out during the procedure. When applying the filtering procedure to the Navier-Stokes equation, we obtain:

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u}_i}{\partial x_k \partial x_k} \quad \text{with} \quad \frac{\partial \overline{u}_i}{\partial x_i} = 0, \tag{2.1}$$

where $\overline{u}_i(\mathbf{x}, t)$ and $\overline{p}(\mathbf{x}, t)$ are the large-scale velocity and pressure fields, and ν is the kinematic viscosity of the fluid. It is common practice to discretize the equations (2.1) on a mesh with grid spacing comparable to Δ since $\overline{\mathbf{u}}(\mathbf{x}, t)$ varies smoothly over Δ.

The subgrid scale stress tensor $\tau_{ij}(\mathbf{x}, t) \equiv \overline{u_i(\mathbf{x}, t) u_j(\mathbf{x}, t)} - \overline{u}_i(\mathbf{x}, t) \overline{u}_j(\mathbf{x}, t)$ is difficult to model as it represents the effect of the unresolved scales on the resolved ones. In LES the tensor $\tau_{ij}(\mathbf{x}, t)$ needs to be expressed in terms of the grid-scale velocity field $\overline{\mathbf{u}}(\mathbf{x}, t)$ (and its derivatives) in order to close eq. 2.1 (see Lesieur 1997).

Eddy-viscosity models parametrize the SGS stress tensor as

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2\nu_T \overline{S}_{ij} \quad \text{where} \quad \overline{S}_{ij}(\mathbf{x}, t) \equiv \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j}(\mathbf{x}, t) + \frac{\partial \overline{u_j}}{\partial x_i}(\mathbf{x}, t)\right), \tag{2.2}$$

where $\nu_T(\mathbf{x}, t)$ is the scalar eddy-viscosity and $\overline{S}_{ij}$ is the resolved rate-of-strain tensor. In this report we will use the standard, misleading, practice of defining $|\overline{S}| = \sqrt{2S_{ij}S_{ij}}$. This is an empirical modelization based on the idea that SGS motions are primarily responsible for a diffusive transport of momentum from the rapid to the slow grid-scale flow regions. The mathematical basis for the introduction of an eddy-viscosity is not well justified, but, it appears to be workable practice, as advocated by Kraichnan (1976). In this framework the eddy viscosity $\nu_T(\mathbf{x}, t)$ is meant to ensure the correct mean drain of kinetic energy from the grid-scale flow to the SGS motions: $-\langle \tau_{ij} \overline{S}_{ij} \rangle$ from the equations (2.1). Obviously observation is that $\nu_T(\mathbf{x}, t)$ should vanish in any laminar flow-regions.

2.2. *Phenomenological justification of the model*

The model to be tested here can be understood in terms of a simple dimensional argument based on the Navier-Stokes equations and of standard turbulence phenomenology. The next section provides a more accurate justification of the model.

Turbulence in the presence of a homogeneous shear can be described in terms of the standard Navier-Stokes equation decomposed in terms of a fluctuating part of the velocity u' superimposed to an average mean profile (whose constant gradient constitute the shear magnitude $\mathcal{S} = dU/dx$).

In dimensional form, the Refined Kolmogorov Similarity Hypothesis (RKSH) can be written as a balance between the non-linear term of the Navier-Stokes equations and the energy dissipation at the same scale, $\delta u_\Delta^3/\Delta + \alpha \cdot \mathcal{S}\delta u_\Delta^2 \sim \varepsilon_\Delta$. The above statistical relation is of course true for each scale Δ in the inertial range, and in the context of LES, this can be thought as the cutoff, filtering scale. In the above relation one can notice that, with respect to the usual form of the RKSH, one has a second term, still coming from the non-linear term of the Navier-Stokes equation and proportional to the shear magnitude.

A simple way to estimate the energy dissipation ε_Δ is through the gradients at the dissipative scale $\varepsilon = \nu\,(\delta u(\eta)/\eta)^2$ and in the context of LES, $\varepsilon_\Delta = \nu_T\,(\delta u(\Delta)/\Delta)^2$, which can be read as the "definition" of the eddy-viscosity, ν_T. Combining the two relations, one obtains $\delta u_\Delta^3/\Delta + \alpha \cdot \mathcal{S}\delta u_\Delta^2 \sim \nu_T\,(\delta u_\Delta/\Delta)^2$ from which $\nu_T = \Delta \cdot (\delta u_\Delta/\Delta + \alpha\mathcal{S})$. Here we note that the above arguments are based on dimensional estimates and for this reason we may be missing an overall $\alpha = \mathcal{O}(1)$ factor in between the two terms. The final expression for the eddy viscosity will be

$$\nu_T = (C_S\Delta)^2 \cdot (|S_\Delta| - |\mathcal{S}|)\,. \tag{2.3}$$

The above expression is the final form for the Shear Improved Smagorinsky Model (SISM) eddy viscosity, $|S_\Delta|$ and $|\mathcal{S}|$ are, respectively, the norms of the resolved strain and of the shear; C_S is an $\mathcal{O}(1)$ constant (Smagorinsky constant) that dimensional arguments cannot determine. We have also posed $\alpha = -1$ based on the physical requirement that for laminar flow the eddy viscosity should vanish.

2.3. *Formulation of the model*

This section explains the definition of the Shear Improved Smagorinsky Model (SISM) in greater details, similarly to what was done in Leveque *et al.* (2006). The theoretical basis of our model was put forward in Toschi *et al.* (2000) on account of previous numerical and experimental studies on wall-bounded turbulence (see Benzi *et al.* 1999, Toschi *et al.* 1999, Ruiz-Chavarria *et al.* 2000). For simplicity, we consider the case of turbulence in a region where it may be reasonably approximated as a statistically stationary homogeneous shear flow (see Hinze 1975).

For a homogeneous shear flow, the velocity field $\mathbf{u}(\mathbf{x},t)$ can be decomposed in a fluctuating and non-fluctuating components $u_i(\mathbf{x},t) = u_i'(\mathbf{x},t) + (\partial U_i/\partial x_j)x_j$, here $U_i(\mathbf{x})$ and $u_i'(\mathbf{x},t)$ denote the mean and fluctuating part of the velocity, respectively. Starting from the exact dynamical equations for the two-point correlation function $R(\mathbf{r}) = \langle u_i'(\mathbf{x},t)u_i'(\mathbf{x}+\mathbf{r},t)\rangle$ and by integrating that equation over a sphere B_r of radius r centered at $\mathbf{x}$, one can write an energy budget (at scale r) that takes the form (see Casciola *et al.* 2003 Danaila *et al.* 2004):

$$S_3^{\mathrm{tr}}(r) + S_3^{\mathrm{pr}}(r) = -\frac{4}{3}\varepsilon r + 2\nu\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{4\pi r^2}\oint_{\partial B_r}\langle|\delta\mathbf{u}'(\mathbf{x},\mathbf{r},t)|^2\rangle\,\mathrm{dS}\right). \tag{2.4}$$

The two contributions $S_3^{\mathrm{tr}}(r)$ and $S_3^{\mathrm{pr}}(r)$ come from the non-linear term of the Navier-Stokes equations; here TR and PR stand for transfer and production, respectively. The right-hand side of (2.4), ε denotes the mean rate of energy dissipation and the second term encompasses finite-Reynolds-number effects (at scales larger than r) with $\delta\mathbf{u}'(\mathbf{x},\mathbf{r},t) \equiv \mathbf{u}'(\mathbf{x}+\mathbf{r},t) - \mathbf{u}'(\mathbf{x},t)$. The energy budget (2.4) is a generalization of the Karman-Howarth equation for the case of homogeneous shear turbulence (see Hinze 1975). In the framework of LES, it may also be interpreted as the mean SGS energy budget with respect to the grid scale r. More explicitly,

$$S_3^{\rm tr}(r) = \frac{1}{4\pi r^2} \oint_{\partial B_r} \left(\langle |\delta \mathbf{u}'(\mathbf{x},\mathbf{r},t)|^2 \delta u_i'(\mathbf{x},\mathbf{r},t)\rangle + \frac{\partial U_i}{\partial x_j} r_j \langle |\delta \mathbf{u}'(\mathbf{x},\mathbf{r},t)|^2\rangle \right) \mathrm{dS}_i \qquad (2.5)$$

represents the transfer of kinetic energy from grid-scale motions (at scales larger than r) to subgrid-scale motions. Also, it indicates that this transfer results from both the grid-scale turbulent fluctuations and the mean shear. These two effects correspond respectively to the non-linear triple-correlation term and the rapid (or linear) term entering in the spectral decomposition of the energy transfer. The second term in the left-hand side of Eq. (2.4) represents a production of SGS kinetic energy induced by the mean shear, and is expressed as

$$S_3^{\rm pr}(r) = \frac{1}{4\pi r^2} \int_{B_r} 2 \frac{\partial U_i}{\partial x_j} \langle \delta u_i'(\mathbf{x},\mathbf{r},t) \delta u_j'(\mathbf{x},\mathbf{r},t)\rangle \mathrm{dV}. \qquad (2.6)$$

In brief, the budget (2.4) means that SGS fluctuations are sustained against molecular dissipation (represented by the right-hand side) by the transfer of energy from grid-scale motions ($S_3^{\rm tr}(r)$) and the energy production induced directly by the mean shear ($S_3^{\rm pr}(r)$).

The estimate of the SGS energy flux is of primary importance in the modeling of the eddy-viscosity. In the previous budget, this flux should be identified with $S_3^{\rm tr}(\Delta)/\Delta$. One may generally consider that turbulent grid-scale velocity differences, $\delta u'(\Delta)$, typically behave as the fluctuating part of the resolved rate-of-strain $|\overline{S'}|$ multiplied by Δ: $\delta u'(\Delta) \approx |\overline{S'}|\Delta$ and similarly, $\delta U(\Delta) \approx |\langle \overline{S}\rangle|\Delta$. The exact expression (2.5) therefore suggests that the mean SGS energy flux should involve two separate contributions, respectively, of order $\Delta^2 \langle |\overline{S'}|^3\rangle$ and $\Delta^2 |\langle \overline{S}\rangle| \langle |\overline{S'}|^2\rangle$.

The Shear Improved Smagorinsky Model (SISM), as defined in Leveque *et al.* (2006), reads as follows:

$$\nu_T(\mathbf{x},t) = (C_s\Delta)^2 \cdot \left(|\overline{S}(\mathbf{x},t)| - |\langle \overline{S}(\mathbf{x},t)\rangle| \right) \qquad (2.7)$$

where the angle brackets $\langle\ \rangle$ denote an ensemble average, a space average over homogeneous directions and/or a time average (discussed again in Section 4). From definition (2.7) it follows that the modeled mean SGS energy flux reads:

$$F_{\rm sgs} \equiv -\langle \tau_{ij} \overline{S}_{ij}\rangle = (C_s\Delta)^2 \ \left(\langle |\overline{S}|^3\rangle - |\langle \overline{S}\rangle| \langle |\overline{S}|^2\rangle \right). \qquad (2.8)$$

We notice that, straightforwardly, this flux vanishes if the resolved turbulence disappears, i.e., if $\overline{S} = \langle \overline{S}\rangle$.

In flow regions where $|\overline{S'}| \gg |\langle \overline{S}\rangle|$, the SGS energy flux is expected to reduce to the contribution of order $\Delta^2 \langle |\overline{S'}|^3\rangle$. In these regions, the mean shear is too weak to perturb the grid-scale dynamics; eddies of size comparable to the grid-scale Δ adjust dynamically via non-linear interactions to transfer energy to SGS motions. This is the standard mechanism behind homogeneous and isotropic turbulence (see Frisch 1996). From expression (2.8) the Smagorinsky estimate $F_{\rm sgs} \simeq (C_s\Delta)^2 \langle |\overline{S'}|^3\rangle$ is consistently recovered in that case. In regions where $|\langle \overline{S}\rangle| \gg |\overline{S'}|$, the behavior of the flow is different: eddies of size comparable to the grid-scale have no time to adjust dynamically and are rapidly distorted by the mean shear (see Liu *et al.* 1999). In these regions, the SGS energy flux is driven by the mean shear and therefore is dominated by the contribution of order $\Delta^2 |\langle \overline{S}\rangle| \langle |\overline{S'}|^2\rangle$. From expression (2.8) and assuming that $\langle |\overline{S}|^3\rangle \approx \langle |\overline{S}|^2\rangle^{3/2}$, one obtains $F_{\rm sgs} \simeq 1/2\ (C_s\Delta)^2 |\langle \overline{S}\rangle|\ \langle |\overline{S'}|^2\rangle$ in agreement with the previous argument. This supports the idea that the expression (2.7) for the eddy-viscosity is consistent with the SGS energy budget of (locally homogeneous) shear turbulence in the both limiting

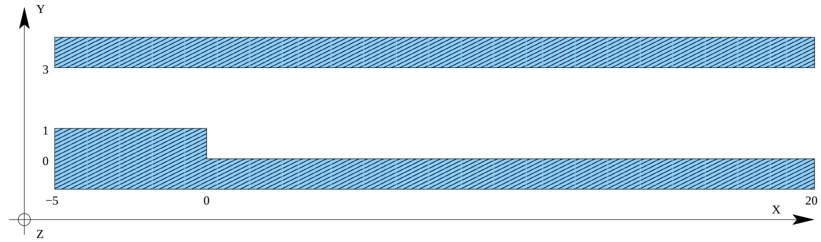

FIGURE 1. Backward-facing step flow geometry; the flow is from left to right. Shaded regions represent impenetrable wall boundaries. The grid is locally refined at the edge of the step. The grid is linearly stretched with local factors 4 at $x = -5$, 4 at $x = -1$, 1 at $x = 0$, 2 at $x = 2$, 2 at $x = 10$, and 4 at $x = 20$. In the vertical direction factors were 1 at $y = 0$, 10 at $y = 0.5$, 1 at $y = 1$, 20 at $y = 2$, 1 at $y = 3$.

cases $|\overline{S'}| \gg |\langle \overline{S} \rangle|$ and $|\langle \overline{S} \rangle| \gg |\overline{S'}|$. The first results concerning plane-channel flows (see Leveque *et al.* 2007) indicate that the model actually abridges well between these two situations without the need for any additional adjustment.

Here we stress that the formulation the SISM SGS model exhibits similarities with the model originally proposed by Schumann (1975), based on a two-part eddy viscosity accounting for the interplay between the non-linear energy cascade present in isotropic turbulence and mean shear effects associated with anisotropy. However, it clearly differs from Schumann's proposal, which additionally, requires an empirical prescription for the "inhomogeneous eddy-viscosity". Note that the simpler formulation $\nu_T = (C_s\Delta)^2|\overline{S} - \langle \overline{S} \rangle|$, as suggested for example in Berselli *et al.* (2005), would lead to $F_{\text{sgs}} \simeq \ (C_s\Delta)^2|\langle \overline{S} \rangle|^2\langle|\overline{S'}|\rangle$ when $|\langle \overline{S} \rangle| \gg |\overline{S'}|$, which would contradict the previous calculation of the mean SGS energy flux.

2.4. *General consideration on the SISM model*

Some of the properties of the SISM model can be deduced directly from its formulation. First, it reduces to the standard Smagorinsky model when no mean shear is present (a common situation away from wall boundaries). In homogeneous and isotropic turbulence, the Smagorinsky model is know to perform reasonably well. Second, the eddy viscosity predicted by the SISM model vanishes in laminar-flow regions, a clear improvement over the standard Smagorinsky model, which would produce an artificial finite value for the eddy viscosity as long as shear is present. Third, the subtraction of the shear from the strain helps to suppress the viscosity close to boundaries. Even if the eddy viscosity does not exactly go to zero at the wall (but rather approaches a finite value), its decreased magnitude allows the model not to be as over-dissipative as the standard Smagorinsky model.

These advantageous properties led to accurate prediction of turbulent channel flow by Leveque *et al.* (2006). To test the model in a more complex case, that also includes flow separation and reattachment, we will now apply it to the calculation of a backward-facing step.

3. Test of the model

3.1. *Details of the backward-facing step case*

We tested the SISM model in a backward-facing step flow configuration with resolution $256 \times 96 \times 64$ in the x, y, and z- directions, respectively (x is the streamwise direction,

y the one normal to the walls, z the spanwise). The Reynolds number based on the step height h and bulk velocity U_b was 4800. This value is close to 4775 in the experiment by Kasagi and Matsunaga (1995); Reynolds number Re_H based on the step height and a centerline velocity at the inlet is 5500. The domain depth in z-direction is 3. The grid used for the simulation was locally refined by factors 10 ($0 < y < 1$) and 20 ($1 < y < 3$) in y-direction and with an expansion factor of up to 4 from the backstep edge to the inflow/outflow regions (Fig. 1).

Typical runs were performed for $100,000$ time steps, with an inflow condition coming from a channel flow simulation at $Re_\tau = 290$. The time step is $0.01h/U_b$. A convective condition is applied at the outflow boundary (Lowery and Reynolds 1986). This simulation was performed using JetCode, an incompressible-flow solver developed at Stanford University and based on a second-order staggered grid discretization, a second-order time advancement, and a Poisson equation for pressure (see Akselvoll and Moin 1995, Pierce 2001).

We performed runs using the standard values of the Smagorinsky constant as commonly accepted for homogeneous and isotropic turbulence ($C_S = 0.2$ see Clark *et al.* 1979) and we implemented the SISM model as defined in Eq. (2.3). Our production runs were carried out using eight CPUs of the Cray XD1 machine at CINECA (Bologna, Italy). In addition to the SISM runs we also performed comparison simulations using the dynamical Smagorinsky model (DSM) (see Germano *et al.* 1991, Pierce and Moin 1998, Lilly 1992, Moin *et al.* 1991) and the standard Smagorinsky model (SM) (see Smagorinsky 1963).

In our production run the mean shear was obtained by averaging in the direction of homogeneity and then was accumulated over time using a running average with a weight recycling factor comparable to the eddy turnover time of the flow, i.e., $50H/U_x$, which means roughly 5000 time steps. However, varying the time-averaging window did not lead to large changes in the results as the flow is statistically stationary and the mean shear was already averaged in the homogeneous direction.

3.2. *Results*

The results obtained are illustrated in Figs. 2 and 3, in which we show averaged profiles and low-order statistics for the flow. Those quantities are compared with the experimental data from Kasagi and Matsunaga (1995) and results from a LES performed using a Dynamical Smagorinsky Model (DSM) with the same geometry, grid, resolution, and Reynolds number. The comparison is performed at several downstream stations in the backstep geometry: in particular measurement stations are placed at positions $x = -2, 0, 2, 4, 6, 8, 10$ and $x = 20$. Here we recall that $x = -2$ is a position on top of the step (still far enough from the inlet; the inflow condition was supplied at $x = -5$) and that $x = 0$ correspond to the edge of the step.

4. Open issues

In Fig. 2 we show the comparison between averaged streamwise velocity profiles. The agreement between the two models and the experiment is excellent. In particular, the location of the reattachment point is predicted correctly (see also Fig. 4, in which the skin-friction coefficient $C_f = 2\tau_w/\rho U_b^2$ is shown). Notice in particular that the strength of the secondary recirculation bubble near the corner (a sensitive diagnostic) is also predicted accurately by the SISM. The small difference in the overall amplitudes can be explained in terms of different flow conditions: our LES has $Re_H \sim 5500$ and expansion ratio 1.5, while Le *et al.* (1997) has $Re_H = 5100$ and expansion ratio 1.2.

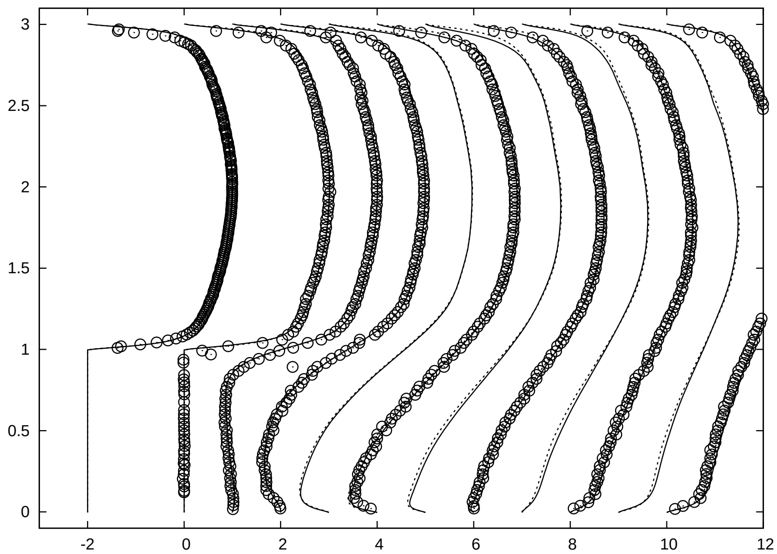

FIGURE 2. Mean streamwise velocity (U) profiles. ——— SISM; - - - DSM; ○ : Experiment Kasagi and Matsunaga (1995).

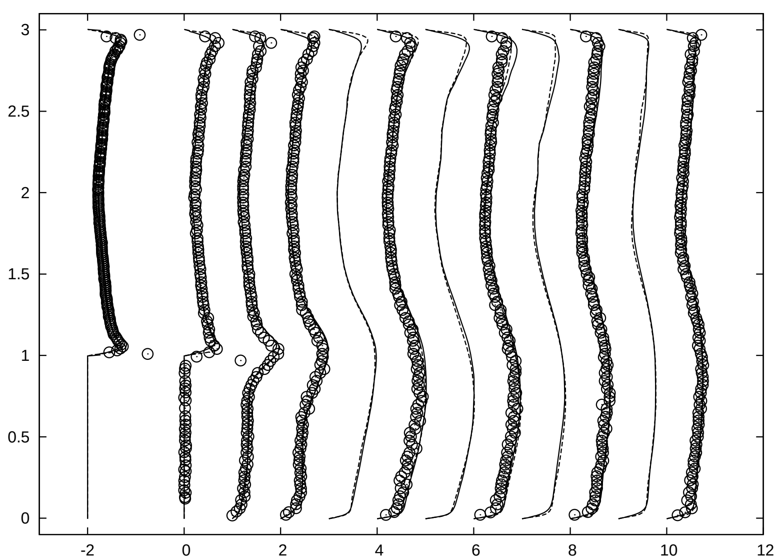

FIGURE 3. Streamwise rms velocity fluctuations u_{rms}, ——— SISM; - - - DSM; ○ Experiment Kasagi and Matsunaga (1995).

Concerning the Reynolds stress components, see Fig. 5, some discrepancy between the SISM and the DSM is visible, but there is still agreement. With respect to the experimental data, it can be considered more than satisfactory.

Finally, in Fig. 6 we show the behavior of the eddy viscosity normalized with the value of the molecular viscosity, at several different stations. For comparison, we have plotted in Fig. 6 the measurement performed with the SISM, the DSM, and the SM. As can be

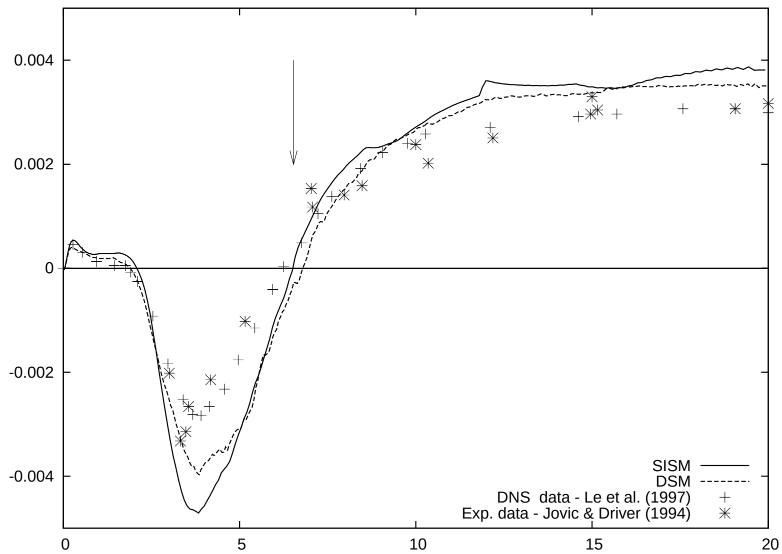

FIGURE 4. Skin-friction coefficient, line style correspond respectively to ——— SISM; - - - DSM. The symbols correspond to DNS and experimental data, respectively from Le *et al.* (1997) and Jovic and Driver (1994).

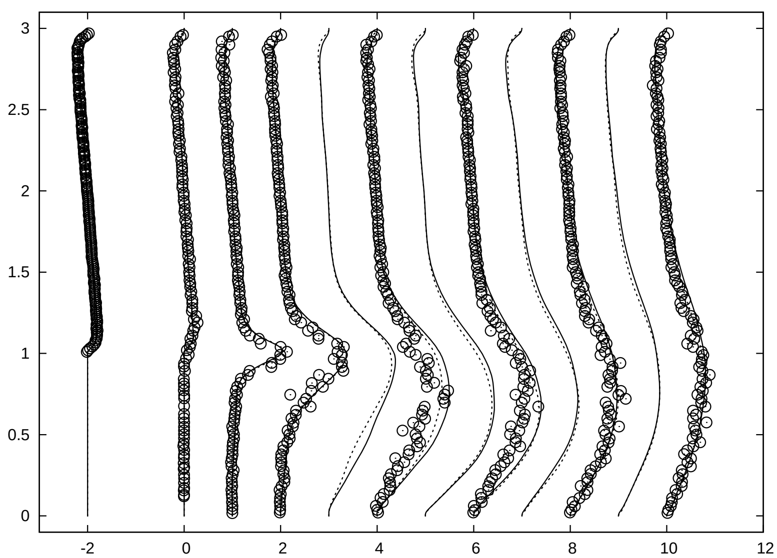

FIGURE 5. Reynolds stress component $\langle uv \rangle$ averaged velocity profiles for: ——— SISM; - - - DSM. Circles represents experimental data from Kasagi and Matsunaga (1995). Curves were stretched by a factor 100 in the x direction for readability.

seen, the SISM and SM give a viscosity quite similar away from strongly sheared regions, as it should. Furthermore, the SM has a huge peak of the viscosity approaching the wall, making the model far too dissipative. The SISM model instead presents a much more regular behavior.

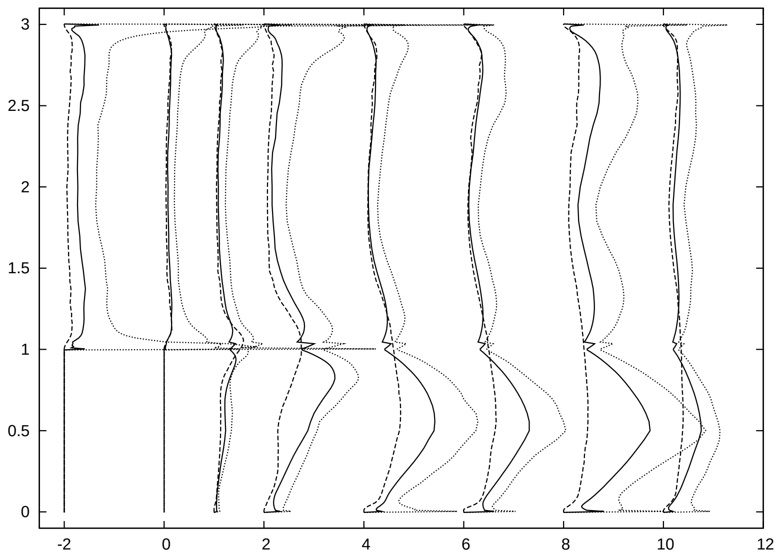

FIGURE 6. Behavior of ν_T/ν at several station for ——— SISM; - - - DSM; ········ SM.

From the results discussed in Section 3.2 we have seen that, overall, the SIMS model performs rather well, at least with respect to the DSM and for this particular type of geometry, resolution, and Reynolds number. These results are comparable to the test performed in Leveque *et al.* (2006). With respect to the DSM, the SISM has the advantage: it is computationally simpler and less expensive. However, there are some evident limitations of the SISM model that should be overcome in order to make the model useful in more general flow condition.

The formulation used for our backward-facing step tests consisted of evaluating the shear at each position in the flow. We found a rapid convergence of the model quite irrespective of the way the shear was accumulated. However, it is evident that in a time non-stationary flow, it would not be wise to keep the shear fixed at changing the underling flow conditions. It is quite evident that for time non-stationary flow, with a shear decreasing in time, this test may result in a negative value for the eddy viscosity if we are unable to promptly adjust the shear level in the definition of the SISM eddy viscosity.

In order to define a proper "instantaneous shear", one needs to average the velocity profile on a proper time window: not too short, to get a stable signal (not just an instantaneous flow profile) and not too long (in such a way to be able to eventually respond to changes integral flow changes). One possibility could be to define the local averaging window by means of the local large-scale eddy turnover time of the flow. How to exactly define this procedure and how solid results will be in those time-dependent situations will be matter of future investigations.

Another future issue will be to check the robustness of the SISM on very coarse grids. We know indeed that in its current definition the SISM has a "small" but non-zero viscosity at the wall. The importance of this residual viscosity will have to be investigated systematically.

5. Conclusions

Recently in Leveque *et al.* (2006) a Shear Improved Smagorinsky model (SISM) was proposed as a candidate eddy-viscosity model for generic flows based on theoretical considerations coming from the physics of shear turbulence, a common feature of any boundary layer turbulence. Here we performed investigations testing the performance of the SISM model in a complex geometry: namely a backward-facing step geometry. Also we compared the results obtained with the SISM against results from standard Smagorinsky and dynamic Smagorinsky models. The standard Smagorinsky model performed very poorly with severe deviation already at the level of averaged streamwise flow profiles (not shown). The quality of the average profiles and low order statistics emerging from the SISM model are acceptable with respect to experimental data and almost indistinguishable from the coming from the dynamical model. The presently studied model has the advantage over the DSM due to an extra computational simplicity and lower cost. Nonetheless, work remains for the future and here we have identified some open issues that will have to be addressed in order to extend the applicability of the model to the most general flow conditions.

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